

Electroweak Symmetry Breaking in Supersymmetric Gauge-Higgs Unification Models

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Abstract

We examine the Higgs mass parameters and electroweak symmetry breaking in supersymmetric orbifold field theories in which the 4-dimensional Higgs fields originate from higher-dimensional gauge supermultiplets. It is noted that such gauge-Higgs unification leads to a specific boundary condition on the Higgs mass parameters at the compactification scale, which is independent of the details of supersymmetry breaking mechanism. With this boundary condition, phenomenologically viable parameter space of the model is severely constrained by the condition of electroweak symmetry breaking for supersymmetry breaking scenarios which can be realized naturally in orbifold field theories. For instance, if it is assumed that the 4-dimensional effective theory is the minimal supersymmetric standard model with supersymmetry breaking parameters induced mainly by the Scherk-Schwarz mechanism, a correct electroweak symmetry breaking can not be achieved for reasonable range of parameters of the model, even when one includes additional contributions to the Higgs mass parameters from the auxiliary component of 4-dimensional conformal compensator. However if there exists a supersymmetry breaking mediated by brane superfield, sizable portion of the parameter space can give a correct electroweak symmetry breaking.

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I. INTRODUCTION

It has been noticed that theories with extra dimension can provide an elegant mechanism to generate various hierarchical structures in 4-dimensional (4D) physics, e.g. the scale hierarchy $M_W/M_{Pl} \approx 10^{-16}$ [1] or the doublet-triplet splitting in grand unified theories [2]. Another interesting possibility with extra dimension is that 4D Higgs fields originate from higher-dimensional gauge fields, unifying Higgs fields with gauge fields [3, 4]. However, constructing a realistic model of gauge-Higgs unification is non-trivial because of the difficulties to obtain the Higgs quartic coupling and a realistic form of Yukawa couplings [5, 6]. Recently it has been pointed out that the idea of gauge-Higgs unification can be implemented successfully within the framework of supersymmetric orbifold field theories [7–9] in which the Higgs quartic couplings can be given by the usual D -term potential of $SU(2) \times U(1)$, and also a realistic form of Yukawa couplings can be obtained through the quasi-localization of bulk fermions and the mixings with brane fermions.

In this paper, we examine the Higgs mass parameters and the resulting electroweak symmetry breaking in supersymmetric gauge-Higgs unification models. To be specific, we will focus on 5D models, however some of our results are valid in more general cases. One model-independent prediction of supersymmetric gauge-Higgs unification is a specific boundary condition on the Higgs mass parameters at the compactification scale. In fact, the predicted boundary condition is same as the one which has been obtained in supersymmetric pseudo-Goldstone Higgs boson models [10]. This can be easily understood by noting that higher-dimensional gauge symmetry constrains the Kähler potential of Higgs superfields in the same way as non-linear global symmetry in pseudo-Goldstone Higgs boson models does.

Although the boundary condition on the Higgs mass parameters predicted by gauge-Higgs unification is independent of the details of supersymmetry (SUSY) breaking, its phenomenological viability severely depends on SUSY breaking mechanism since the Higgs mass parameters at the weak scale receive large radiative corrections depending on other sparticle masses [11]. It turns out that the condition of electroweak symmetry breaking severely restricts the parameter space of the model for SUSY breaking scenarios which can be realized naturally in orbifold field theories. If $N = 1$ SUSY breaking masses are much smaller than the compactification scale, which will be assumed throughout this paper, the $N = 1$ SUSY breaking can be described by the auxiliary components of 4D $N = 1$ superfields. Then

there can be two distinctive sources of SUSY breaking for visible fields, one mediated by the zero modes of bulk superfields propagating in 5D spacetime and the other mediated by 4D brane superfields confined on the orbifold fixed points. An attractive possibility is that SUSY breaking is mediated dominantly by the bulk radion superfield T , which is equivalent to the Scherk-Schwarz (SS) SUSY breaking by boundary conditions [12, 13]. Another source of SUSY breaking in bulk is the auxiliary component of 4D supergravity (SUGRA) multiplet which can be parameterized by the F -component of the chiral conformal-compensator superfield Ω [14]. Classical conformal invariance ensures that soft scalar masses, gaugino masses and trilinear A -parameters are not affected by F^Ω at tree level, however the Higgs μ and $B\mu$ parameters receive contributions from F^Ω even at tree level [15]. As we will see, if the 4D effective theory of the model is given by the minimal supersymmetric standard model (MSSM) with soft SUSY breaking parameters induced by the F -components of the radion superfield T and the compensator superfield Ω , a correct electroweak symmetry breaking can *not* be achieved for reasonable range of parameters of the model. Therefore one needs an additional SUSY breaking other than those from the SS mechanism and the 4D SUGRA multiplet, e.g. the SUSY breaking mediated by a brane superfield, in order to achieve electroweak symmetry breaking in 5D gauge-Higgs unification models whose 4D effective theory corresponds to the MSSM. When there exists a supersymmetry breaking mediated by brane superfield, sizable portion of the parameter space can give a correct electroweak symmetry breaking.

This paper is organized as follows. In Section II, we discuss SUSY breaking masses in 5D gauge-Higgs unification models, particularly the Higgs mass parameters, in the framework of 4D effective action in $N = 1$ superspace. In Section III, we perform a numerical analysis for electroweak symmetry breaking at the weak scale under the assumption that the 4D effective theory below the compactification scale is given by the MSSM. Section IV is the conclusion.

II. SUSY BREAKING MASSES IN 5D GAUGE-HIGGS UNIFICATION MODELS

The most efficient way to compute SUSY breaking masses is to derive the 4D effective action in $N = 1$ superspace which contains the SUSY-breaking messenger superfields explicitly [14]. In this section, we derive the (part of) 4D effective action of a 5D theory compacti-

fied on $S^1/Z_2 \times Z'_2$ in which the 4D Higgs fields originate from 5D vector supermultiplets, and discuss SUSY breaking masses induced by the auxiliary components of the 4D SUGRA multiplet, the radion superfield, and also generic chiral brane superfields.

In $N = 1$ superspace, a 5D vector multiplet is represented by a vector superfield V and a chiral superfield Σ . The orbifold boundary conditions of these $N = 1$ superfields are given by

$$\begin{aligned} V^a(-y) &= z_a V^a(y), & V^a(-y') &= z'_a V^a(y'), \\ \Sigma^a(-y) &= -z_a \Sigma^a(y), & \Sigma^a(-y') &= -z'_a \Sigma^a(y'), \end{aligned} \quad (1)$$

where $y' = y - \pi$, $z_a = \pm 1$ and $z'_a = \pm 1$. In gauge-Higgs unification models, 4D gauge bosons originate from V^a with $z_a = z'_a = 1$, while 4D Higgs bosons originate from Σ^a with $z_a = z'_a = -1$. The 5D gauge transformations associated with $\{V^a, \Sigma^a\}$ are given by

$$e^V \rightarrow e^\Lambda e^V e^{\Lambda^\dagger}, \quad \Sigma \rightarrow e^\Lambda (\Sigma - \sqrt{2} \partial_y) e^{-\Lambda}, \quad (2)$$

where $V = V^a T^a$, $\Sigma = \Sigma^a T^a$, and $\Lambda = \Lambda^a T^a$ denotes the chiral gauge transformation superfield satisfying the orbifold boundary condition:

$$\Lambda^a(-y) = z_a \Lambda^a(y), \quad \Lambda^a(-y') = z'_a \Lambda^a(y').$$

The 5D action of V^a and Σ^a in $N = 1$ superspace is given by [16]

$$S_{\text{bulk}} = \int d^5x \left[\int d^4\theta \frac{1}{g_5^2} \frac{1}{T + T^*} \left(\partial_y V^a - \frac{1}{\sqrt{2}} (\Sigma^a + \Sigma^{a*}) \right)^2 + \int d^2\theta \frac{T}{4g_5^2} W^{a\alpha} W_\alpha^a + \dots \right], \quad (3)$$

where g_5^2 is the (unified) 5D gauge coupling with mass dimension -1 and the radion superfield T is given by

$$T = R + iB_5 + \theta \Psi_5^2 + \theta^2 F^T,$$

where R is the orbifold radius, B_5 and Ψ_5^2 are the fifth-components of the graviphoton B_M and the symplectic Majorana gravitini Ψ_M^i ($i = 1, 2$). Here we limit ourselves to the terms which are bilinear in V^a and Σ^a . In addition to the above bulk action, there can be interactions of V^a and Σ^a at the orbifold fixed points, particularly the interactions with chiral brane superfields which have nonzero SUSY breaking auxiliary components. Those fixed-point interactions are restricted also by the 5D gauge symmetry (2) and 5D general

covariance, and generically given by

$$S_{\text{brane}} = \int d^5x \left[\int d^4\theta \frac{\delta(y)\Delta Y_a(\mathcal{Z}, \mathcal{Z}^*) + \delta(y-\pi)\Delta Y'_a(\mathcal{Z}', \mathcal{Z}'^*)}{g_5^2(T+T^\dagger)^2} \left(\partial_y V^a - \frac{1}{\sqrt{2}}(\Sigma_a + \Sigma_a^*) \right)^2 \right. \\ \left. + \int d^2\theta \frac{1}{4g_5^2} [\delta(y)\omega_a(\mathcal{Z}) + \delta(y-\pi)\omega'_a(\mathcal{Z}')] W^{a\alpha} W_\alpha^a + \dots \right] \quad (4)$$

where \mathcal{Z} and \mathcal{Z}' stand for generic SUSY breaking brane superfields at $y = 0$ and $y = \pi$, respectively.

The 4D effective action of the gauge and Higgs zero modes at the compactification scale can be written as

$$S_{\text{eff}} = \int d^4x \left[\int d^4\theta e^{\Omega+\Omega^\dagger} \left(Y_{H_1} H_1^\dagger H_1 + Y_{H_2} H_2^\dagger H_2 + \gamma_H H_1 H_2 + \gamma_H^* H_1^\dagger H_2^\dagger \right) \right. \\ \left. + \int d^2\theta \frac{1}{4} f_a W^{a\alpha} W_\alpha^a + \dots \right] \quad (5)$$

where the gauge kinetic function f_a is a holomorphic function of $\{\mathcal{Z}_A\} = \{T, \mathcal{Z}, \mathcal{Z}'\}$, while the Higgs wavefunction coefficients Y_{H_i} and γ_H depend on both $\{\mathcal{Z}_A\}$ and $\{\mathcal{Z}_A^*\}$. Here H_1 and H_2 are the two MSSM Higgs superfields originating from

$$\Sigma^1 = -(H_1 + H_2), \quad \Sigma^2 = -i(H_1 - H_2), \quad (6)$$

with $z_{1,2} = z'_{1,2} = -1$, while $W^{a\alpha}$ are the chiral gauge superfields for the zero modes of V^a with $z_a = z_a = 1$. Note that we have introduced the chiral conformal-compensator superfield e^Ω to parameterize the SUSY breaking by the auxiliary component of 4D SUGRA multiplet [14]. It is then straightforward to find

$$Y_{H_1} = Y_{H_2} = \gamma_H = \frac{2\pi}{g_5^2(T+T^*)} \left[1 + \frac{\Delta Y_H(\mathcal{Z}, \mathcal{Z}', \mathcal{Z}^*, \mathcal{Z}'^*)}{\pi(T+T^*)} \right], \\ f_a = \frac{1}{g_5^2} [\pi T + \omega_a(\mathcal{Z}) + \omega'_a(\mathcal{Z}')] , \quad (7)$$

where ΔY_H represents the contributions from ΔY_a and $\Delta Y'_a$ in (4) for Σ^a from which the Higgs fields originate.

A simple dimensional analysis suggests that the vacuum expectation values of $\omega_a, \omega'_a, \Delta Y_a$ and $\Delta Y'_a$ are generically of the order of the cutoff length scale, and thus

$$\frac{\Delta Y_H(\mathcal{Z}, \mathcal{Z}', \mathcal{Z}^*, \mathcal{Z}'^*)}{\pi(T+T^*)} = \mathcal{O}(1/\pi R\Lambda), \\ \frac{\omega_a(\mathcal{Z}) + \omega'_a(\mathcal{Z}')}{\pi T} = \mathcal{O}(1/\pi R\Lambda), \quad (8)$$

where Λ denotes the cutoff mass scale. By construction, $\pi R\Lambda$ should be bigger than the unity, however its precise value depends on the radion stabilization mechanism in the theory. Throughout this paper, we will assume that the theory is strongly coupled at Λ [17], and also

$$\pi R\Lambda = \mathcal{O}(8\pi^2), \quad (9)$$

which are consistent with each other. Under these assumptions, the contributions from brane actions to the SUSY-preserving components of Y_H and f_a are suppressed by $\mathcal{O}(1/8\pi^2)$ compared to the bulk contributions. Note that this does *not* mean that SUSY-breaking masses are dominated also by the bulk contributions since the F -components of \mathcal{Z} and/or \mathcal{Z}' can be significantly bigger than the F -components of T and Ω .

One important consequence of gauge-Higgs unification is that

$$Y_{H_1} = Y_{H_2} = \gamma_H. \quad (10)$$

Obviously the contributions to $Y_{H_{1,2}}$ and γ_H from the bulk action (3) obey the above relation due to the 5D gauge symmetry (2) unifying the Higgs fields with gauge fields. Generically this bulk gauge symmetry is broken down to a subgroup at the orbifold fixed point by boundary conditions, so one may expect the relation (10) is broken by the contributions from the brane action (4). However still (4) is constrained by (2), in particular, by the non-linear gauge symmetry under which

$$\begin{aligned} \Sigma^1 &\rightarrow \Sigma^1 + \sqrt{2}\partial_y \Lambda^1 + \dots, \\ \Sigma^2 &\rightarrow \Sigma^2 + \sqrt{2}\partial_y \Lambda^2 + \dots, \\ V^{1,2} &\rightarrow V^{1,2} + \Lambda^{1,2} + (\Lambda^{1,2})^\dagger + \dots, \end{aligned} \quad (11)$$

where $\Lambda^{1,2}(-y) = -\Lambda^{1,2}(y)$ and $\Lambda^{1,2}(-y') = -\Lambda^{1,2}(y')$ are parity-odd gauge transformation superfields, and the ellipses denote the transformations not relevant for constraining the terms which are bilinear in the Higgs superfields. This non-linear gauge symmetry constrains the brane actions at the fixed points in such a way that the contributions to $Y_{H_{1,2}}$ and γ_H from the brane actions satisfy the relation (10). Note that the non-linear gauge symmetry (11) is realized for 5D fields, so *not* valid if the massive Kaluza-Klein modes are integrated out. As a result, the relation (10) is valid *only* at the compactification scale, but is modified by radiative corrections at lower energy scales.

Let us discuss SUSY breaking scenarios possible in 5D orbifold field theories. Since we are assuming that SUSY breaking scale is much lower than the compactification scale, any SUSY breaking can be described by the auxiliary components of 4D messenger superfields. In 5D models under consideration, possible messenger superfields include the 4D SUGRA multiplet, the radion superfield T and also some set of brane superfields $\{\mathcal{Z}, \mathcal{Z}'\}$. In the compensator formulation of 4D SUGRA, SUSY breaking by the 4D SUGRA multiplet can be described by the F -component of the chiral conformal-compensator superfield [14]

$$e^\Omega = e^{\Omega_0}(1 + \theta^2 F^\Omega). \quad (12)$$

In the Einstein frame, $\Omega_0 = K/6$ where K is the Kähler potential. Then the compensator F -component is given by

$$F^\Omega = m_{3/2}^* + \frac{1}{3} \frac{\partial K}{\partial \mathcal{Z}_A} F^A, \quad (13)$$

where $m_{3/2} = e^{K/2}W$ is the gravitino mass for the superpotential W and F^A is the F -component of $\{\mathcal{Z}_A\} = \{T, \mathcal{Z}, \mathcal{Z}'\}$. Once the Kähler potential K and the superpotential W of 4D effective theory are known, F^A in the Einstein frame is determined to be [18]

$$F^A = -e^{K/2} K^{AB} \left(\frac{\partial W}{\partial \mathcal{Z}_B} + \frac{\partial K}{\partial \mathcal{Z}_B} W \right)^*, \quad (14)$$

where K^{AB} is the inverse of the Kähler metric $K_{AB} = (\partial^2 K / \partial \mathcal{Z}_A \partial \mathcal{Z}_B^*)$.

The SUSY breaking mediated by F^T corresponds to the Sherk-Schwarz SUSY breaking by twisted boundary condition [13], thus is a natural candidate for SUSY breaking in models with extra dimension. For 4D fields originating from 5D bulk fields, all soft SUSY breaking masses generically receive a contribution of $\mathcal{O}(F^T/R)$ at tree approximation. As for the SUSY breaking mediated by F^Ω , *only* the Higgsino mass μ and the bilinear Higgs coefficient B can receive a contribution of the order of F^Ω at tree level [18], while the gaugino masses M_a , soft scalar masses m_ϕ , and trilinear scalar coefficients A get the conformal anomaly-mediated contributions of $\mathcal{O}(F^\Omega/8\pi^2)$ [15]. The anomaly-mediated scenario corresponds to the case that $F^\Omega/8\pi^2 \gg F^T/R$, so M_a , m_ϕ and A are dominated by the conformal anomaly-mediated contributions. However the anomaly-mediated scenario is not acceptable in gauge-Higgs unification model since μ and B always receive a contribution of $\mathcal{O}(F^\Omega)$, thus become $\mathcal{O}(8\pi^2 M_a)$ unless an unnatural cancellation is assumed, which would be too large to allow a correct electroweak symmetry breaking. In this paper, we will assume that F^Ω is of

the order of F^T/R or less, and thus ignore the conformal anomaly-mediated contributions of $\mathcal{O}(F^\Omega/8\pi^2)$.

Following the standard method to compute soft SUSY breaking masses in 4D SUGRA [18], we find that the gaugino and Higgs mass parameters for generic f_a and Y_H are given by

$$\begin{aligned}\mathcal{L}_{\text{soft}} = & -\left(\frac{1}{2}M_a\lambda_a\lambda_a + \mu\tilde{H}_1\tilde{H}_2 + \text{h.c.}\right) \\ & -(|\mu|^2 + m_{H_1}^2)|H_1|^2 - (|\mu|^2 + m_{H_2}^2)|H_2|^2 + (B\mu H_1 H_2 + \text{h.c.})\end{aligned}\quad (15)$$

where

$$\begin{aligned}M_a &= -\frac{1}{2\text{Re}(f_a)}F^A\frac{\partial f_a}{\partial \mathcal{Z}_A}, \\ \mu^* &= -F^\Omega - F^A\frac{\partial \ln Y_H}{\partial \mathcal{Z}_A}, \\ m_{H_1}^2 + |\mu|^2 &= m_{H_2}^2 + |\mu|^2 = -B\mu = |\mu|^2 - F^A F^{B*} \frac{\partial^2 \ln Y_H}{\partial \mathcal{Z}_A \partial \mathcal{Z}_B^*},\end{aligned}\quad (16)$$

for *canonically normalized* gauginos λ_a , Higgs bosons H_1 and H_2 , and Higgsinos \tilde{H}_1 and \tilde{H}_2 . Note that the relation between Higgs mass parameters

$$m_{H_1}^2 + |\mu|^2 = m_{H_2}^2 + |\mu|^2 = -B\mu \quad (17)$$

is a consequence of (10), thus can be considered as a prediction of gauge-Higgs unification which is independent of the details of SUSY breaking mechanism. As we have noticed, (10) is a consequence of the non-linear 5D gauge symmetry (11) which is valid only at scales above the compactification scale, so the above relation corresponds to a boundary condition at the compactification scale. In fact, the same relation between Higgs mass parameters has been obtained before in the context of supersymmetric pseudo-Goldstone Higgs models. This is not surprising because the non-linear 5D gauge symmetry in gauge-Higgs unification models plays the same role as the non-linear global symmetry in pseudo-Goldstone Higgs models.

One attractive way to break SUSY in orbifold field theory is the Scherk-Schwarz mechanism to impose different boundary conditions for different fields in the same supermultiplet [12]. It has been pointed out that the SS breaking is equivalent to the SUSY breaking by F^T [13]. Here we consider a more generic situation that there exist additional contributions from F^Ω to the Higgs mass parameters, which are generically of $\mathcal{O}(F^T/R)$. In this SUSY

breaking scenario mediated by F^T and F^Ω , the gaugino masses and Higgs mass parameters are given by

$$\begin{aligned} M_a &= -\frac{F^T}{2R}, \\ \mu^* &= -F^\Omega + \frac{F^T}{2R}, \\ m_{H_1}^2 + |\mu|^2 &= m_{H_2}^2 + |\mu|^2 = -B\mu = -|M_a|^2 + |\mu|^2, \end{aligned} \quad (18)$$

where we have ignored the contributions from brane actions under the assumption of (8) and (9), and also the loop-suppressed anomaly-mediated contributions of $\mathcal{O}(F^\Omega/8\pi^2)$.

To study the electroweak symmetry breaking, one needs also information on the sfermion masses and trilinear A -parameters for the quark and lepton superfields having large Yukawa coupling. In fact, the sfermion masses and A -parameters (at the compactification scale) in gauge-Higgs unification model depend highly on the details of the mechanism to generate hierarchical Yukawa couplings and flavor mixings, i.e. on the details of the quasi-localization of bulk fermions and the mixings with brane fermions. However, to have a large top-quark Yukawa coupling $y_t \approx g_{GUT}$, there should not be any sizable suppression of the top-quark Yukawa coupling by quasi-localization and/or the mixing with brane fermions. In this case, the Kähler metrics of the $SU(2)$ -doublet top quark superfield Q_3 and the $SU(2)$ -singlet top quark superfield U_3 are given by [19]

$$\begin{aligned} Y_{Q_3} &\approx \frac{\pi}{2}(T + T^*) + \Delta Y_{Q_3}(\mathcal{Z}, \mathcal{Z}', \mathcal{Z}^*, \mathcal{Z}'^*) \\ Y_{U_3} &\approx \frac{\pi}{2}(T + T^*) + \Delta Y_{U_3}(\mathcal{Z}, \mathcal{Z}', \mathcal{Z}^*, \mathcal{Z}'^*) \end{aligned} \quad (19)$$

where ΔY represent the contributions from brane actions. The resulting soft stop masses and A -parameter are given by

$$\mathcal{L}_{\text{soft}} = -m_{Q_3}^2 |\tilde{Q}_3|^2 - m_{U_3}^2 |\tilde{U}_3|^2 + (A_t y_t H_2 \tilde{Q}_3 \tilde{U}_3 + \text{h.c.}) \quad (20)$$

where

$$\begin{aligned} m_{Q_3}^2 &= -F^A F^{B*} \frac{\partial^2 \ln Y_{Q_3}}{\partial \mathcal{Z}_A \partial \mathcal{Z}_B^*}, \\ m_{U_3}^2 &= -F^A F^{B*} \frac{\partial^2 \ln Y_{U_3}}{\partial \mathcal{Z}_A \partial \mathcal{Z}_B^*}, \\ A_t &= -F^A \frac{\partial \ln(Y_H Y_{Q_3} Y_{U_3})}{\partial \mathcal{Z}_A}. \end{aligned} \quad (21)$$

When SUSY breaking is dominated by F^T and F^Ω , we have

$$\begin{aligned} A_t &\approx \frac{F^T}{2R} = -M_a, \\ m_{Q_3}^2 &\approx m_{U_3}^2 \approx \left| \frac{F^T}{2R} \right|^2. \end{aligned} \quad (22)$$

If $\tan\beta = \langle H_2^0 \rangle / \langle H_1^0 \rangle$ is large, the b -quark and τ -lepton also have a large Yukawa coupling, and then the sbottom and stau have similar soft masses and A -parameters [19].

III. ANALYSIS OF ELECTROWEAK SYMMETRY BREAKING

Given the predictions for soft SUSY breaking masses at the compactification scale, we analyze electroweak symmetry breaking by numerically solving the renormalization group (RG) equations down to the symmetry breaking scale, M_{SB} , and minimizing the effective Higgs potential. We identify the compactification scale as the gauge unification scale, $M_X = 2 \times 10^{16}$ GeV together with an assumption that GUT scale gaugino masses are universal, $M_a(M_X) = M_{1/2}$, and M_{SB} as $\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$ following the standard procedure where $m_{\tilde{t}_{1,2}}$ are the stop mass eigenvalues[20]. It is further assumed that the effective theory between M_X and M_{SB} is given by the MSSM which is reduced to the standard model at scales below M_{SB} . We will use the parameter convention in which the gauge-Higgs unification prediction for Higgs mass parameters takes the form (17).

To obtain the gauge and Yukawa couplings at M_X , we solve two-loop RG running [21] from the electroweak scale to M_X , however the matching at M_{SB} is performed without including the superparticle threshold corrections. Using the boundary conditions at M_X , the Higgs mass parameters at M_{SB} is calculated using one-loop RG equations for dimensionful parameters and two-loop RG equations for the gauge and Yukawa couplings. Given the inputs of dimensionless parameters at the electroweak scale and dimensionful parameters at M_X , a self-consistent value of M_{SB} is extracted by numerical iteration starting from M_{SB} taken to be the top quark mass.

A simple way to analyze the electroweak symmetry breaking is to compute the RG-improved tree-level Higgs potential at the electroweak scale which can be written as

$$V = m_1^2 |H_1^0|^2 + m_2^2 |H_2^0|^2 - (B\mu H_1^0 H_2^0 + \text{c.c.}) + \frac{1}{8}(g_1^2 + g_2^2) (|H_1^0|^2 - |H_2^0|^2)^2, \quad (23)$$

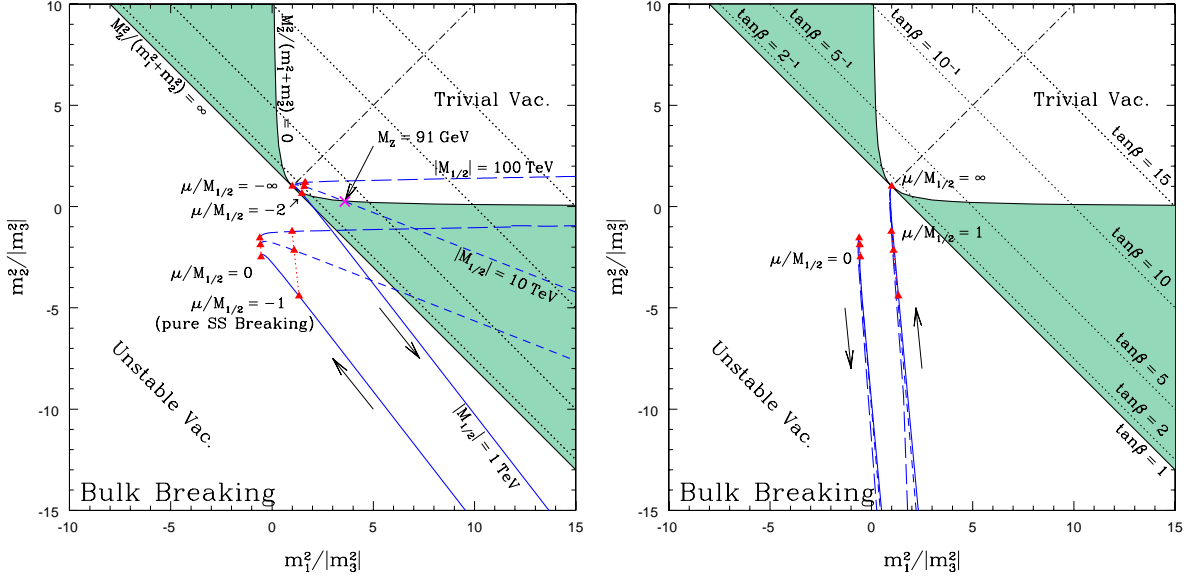


FIG. 1: Results of the analysis of the RG-improved tree level Higgs potential for the bulk SUSY breaking mediated by F^T and F^Ω . Here $m_i^2 = m_{H_i}^2 + |\mu|^2$ ($i = 1, 2$) and $m_3^2 = B\mu$ at M_{SB} . Electroweak symmetry is broken in the shaded regions. The solid, dashed and long-dashed lines indicate the solutions of RG equations for $|M_{1/2}| = 1, 10, 100$ TeV, respectively. The parameter ratio $\mu/M_{1/2}$ is varying from $-\infty$ to 0 in Fig.1a (left), and from 0 to ∞ in Fig.1b (right). Note that the pure Sherk-Schwarz SUSY breaking scenario, i.e. $F^\Omega = 0$, gives $\mu/M_{1/2} = -1$, which is clearly outside the symmetry breaking shaded region. The long arrows show the direction to which $\mu/M_{1/2}$ increases. The stop soft parameters at M_X are chosen to be $A_t/M_{1/2} = -1$ and $m_{Q_3}^2 = m_{U_3}^2 = M_{1/2}^2$ as given in (22), and the top quark Yukawa coupling $y_t(m_t) = 0.98$.

where $m_{1,2}^2 = m_{H_{1,2}}^2 + |\mu|^2$. To develop non-trivial vacuum ($\langle H_{1,2}^0 \rangle \neq 0$) and stabilize the flat direction of the D-term potential, the Higgs mass parameters should satisfy

$$m_1^2 m_2^2 - |B\mu|^2 < 0, \quad m_1^2 + m_2^2 - 2|B\mu| > 0. \quad (24)$$

Under this conditions, M_Z and $\tan \beta$ at the minimum of the potential acquire the following values:

$$M_Z^2 = (m_1^2 + m_2^2) \left[\left\{ 1 + 4 \left(\frac{m_1^2 m_2^2 - |B\mu|^2}{(m_1^2 - m_2^2)^2} \right) \right\}^{-\frac{1}{2}} - 1 \right], \quad \tan \beta = \frac{m_1^2 + m_2^2}{|B\mu|} - \frac{1}{\tan \beta}. \quad (25)$$

This parameter region for stable electroweak symmetry breaking is given by the shaded area in Fig.1.

Let us now consider the SUSY breaking by F^T and F^Ω , i.e. the Sherk-Schwarz SUSY breaking with additional contributions to Higgs mass parameters from the auxiliary component of the 4D SUGRA multiplet. In this scenario, SUSY breaking parameters at the GUT scale are given by (18) and (22). The resulting solutions of RG equations for $|M_{1/2}| = 1$ TeV, 10 TeV and 100 TeV are depicted in Fig.1 by the solid, dashed and long-dashed lines, respectively. For numerical analysis, we use the top quark Yukawa coupling $y_t(m_t) = 0.98$, and also the GUT scale predictions $A_t/M_{1/2} = -1$ and $m_{Q_3}^2 = m_{U_3}^2 = M_{1/2}^2$. For simplicity, we ignored the effects of the b -quark and τ -lepton Yukawa couplings, so the results can be somewhat modified for large $\tan\beta$. In Fig.1a (left), $\mu(M_X)/M_{1/2}$ varies from $-\infty$ to 0, while it varies from 0 to ∞ in Fig.1b (right). For each value of $|M_{1/2}|$, we have two lines since $B\mu(M_{\text{SB}})$ crosses zero, thereby the solution goes to infinity at some value of $\mu(M_X)/M_{1/2}$. As $|M_{1/2}|$ is decreasing from 100 TeV to 1 TeV, the solutions sweep the shaded region of Fig.1a from upper right to lower right, while there exists no solution in the shaded region of Fig.1b. This indicates that $1 \text{ TeV} \lesssim |M_{1/2}| \lesssim 100 \text{ TeV}$ with $\mu/M_{1/2} < 0$ is required to obtain a symmetry breaking vacuum. Still to get the correct value of M_Z , one needs a fine-tuned value of $\mu(M_X)$ for each value of $|M_{1/2}|$.

The pure Sherk-Schwarz SUSY breaking scenario gives (18) and (22) with $F^\Omega = 0$, so $\mu(M_X) = -M_{1/2}$. As can be seen in Fig.1a, such parameter value develops an unstable vacuum, and thus should be excluded. With $F^\Omega \neq 0$, $\mu/M_{1/2}$ can have an arbitrary value in principle. However for the case with $\mu/M_{1/2} > 0$, as can be seen in Fig.1b, the whole parameter space of (18) and (22) develops an unstable vacuum, thus should be excluded also. A stable vacuum with correct value of M_Z can be obtained for $-2 \lesssim \mu(M_X)/M_{1/2} < -1$, but only for an abnormally large gaugino mass

$$|M_{1/2}| \gtrsim 10 \text{ TeV}.$$

In this case, we need an unnatural fine-tuning of $\mu(M_X)$ with an accuracy $\delta\mu/\mu \lesssim 10^{-4}$ in order to get $M_Z = 91 \text{ GeV}$, which is hard to be accepted. We thus conclude that a correct electroweak symmetry breaking can not be achieved in SUSY breaking scenarios mediated by F^T and F^Ω alone,

To confirm the above results, we perform an alternative analysis including the effects of one-loop effective Higgs potential [22, 23] as well as the effects of the b -quark and τ -lepton Yukawa couplings. Here we treat $B\mu$ and μ at M_{SB} as free parameters and evaluate

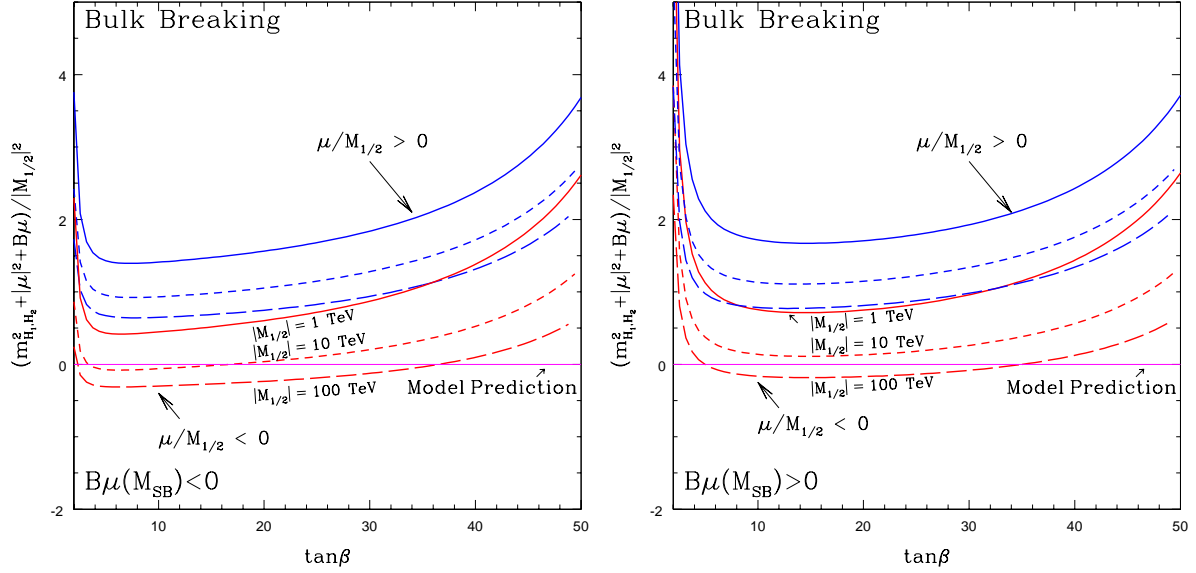


FIG. 2: Results of the alternative analysis for the bulk SUSY breaking mediated by F^T and F^Ω . The value of $(m_{H_1,H_2}^2 + |\mu|^2 + B\mu)/|M_{1/2}|^2$ at M_X is obtained for μ and B at M_{SB} giving $M_Z = 91$ GeV, and is depicted as a function of $\tan\beta$. The solid, dashed and long-dashed curves represent the results for $|M_{1/2}| = 1, 10, 100$ TeV, respectively. The effects of one-loop effective Higgs potential are fully included. The relevant sfermion soft parameters at M_X are assumed to be given by $A_{t,b,\tau}/M_{1/2} = -1$, $m_{Q_3,U_3}^2 = (y_t/g_{GUT})^2 M_{1/2}^2$, $m_{D_3}^2 = (y_b/g_{GUT})^2 M_{1/2}^2$ and $m_{E_3,L_3}^2 = (y_\tau/g_{GUT})^2 M_{1/2}^2$.

their values by minimizing the one-loop corrected effective Higgs potential for fixed values of M_Z , $\tan\beta$ and $|M_{1/2}|$. We then evolve $B\mu$ and μ up to M_X and check if the resulting values satisfy the relation (18). This method works because the RG equations of other soft SUSY breaking masses do not depend on μ or $B\mu$ at one-loop level. For numerical analysis, we use $m_t^{\text{pole}} = 175$ GeV, and the relevant sfermion masses and A -parameters are assumed to be $A_{t,b,\tau}/M_{1/2} = -1$, $m_{Q_3,U_3}^2 = (y_t/g_{GUT})^2 M_{1/2}^2$, $m_{D_3}^2 = (y_b/g_{GUT})^2 M_{1/2}^2$ and $m_{E_3,L_3}^2 = (y_\tau/g_{GUT})^2 M_{1/2}^2$ at M_X , where D_3 , E_3 and L_3 denote the $SU(2)$ -singlet b -quark, $SU(2)$ -singlet τ -lepton, and $SU(2)$ -doublet τ -lepton superfield, respectively. For large $\tan\beta$, which is the case that $A_{b,\tau}$ and m_{D_3,E_3,L_3}^2 become relevant, these forms of $A_{b,\tau}$ and m_{D_3,E_3,L_3}^2

mimic well the actual values [19] for the SUSY breaking by F^T and F^Ω .

The tree-level relations of (25) can be rewritten as

$$\mu^2 = -\frac{M_Z^2}{2} - \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2 \beta}{1 - \tan^2 \beta}, \quad |B\mu| = \frac{\tan \beta}{1 + \tan^2 \beta} (m_{H_1}^2 + m_{H_2}^2 + 2\mu^2). \quad (26)$$

Then the effects of one-loop Higgs potential can be included by replacing $m_{H_{1,2}}^2$ by $m_{H_{1,2}}^2 - t_{1,2}/\langle H_{1,2}^0 \rangle$, where $t_{1,2}$ are given by [24]

$$t_{1,2} = -\frac{1}{32\pi^2} \text{Str} \left[\frac{\partial \mathcal{M}^2}{\partial \langle H_{1,2}^0 \rangle} \mathcal{M}^2 \left(\ln \frac{\mathcal{M}^2}{M_{SB}^2} - 1 \right) \right], \quad (27)$$

for \mathcal{M}^2 representing all the mass matrices in the model. For numerical analysis, we use the form of $t_{1,2}$ given in [25]. Because $t_{1,2}$ depend implicitly on μ through the masses of neutralino, chargino and squarks, we use numerical iteration to evaluate μ starting from a point around the RG-improved tree-level solution.

In Fig.2, we show the result of the analysis computing $m_{H_{1,2}}^2 + \mu^2 + B\mu$ at M_X as a function of $\tan \beta$ for various values of $|M_{1/2}|$. Because the electroweak symmetry breaking condition does not depend on the signs of μ and $B\mu$, we have four different cases distinguished by the signs of μ and μB . Fig.2 shows that the condition of gauge-Higgs unification, i.e. $m_{H_{1,2}}^2 + \mu^2 + B\mu = 0$ at M_X , cannot be satisfied for reasonable range of $|M_{1/2}|$. To satisfy the gauge-Higgs unification condition, $|M_{1/2}| \gtrsim 10$ TeV is required as we have anticipated from the analysis based on the RG-improved tree-level Higgs potential.

We saw that a correct electroweak symmetry breaking is not allowed when SUSY breaking is mediated dominantly by the bulk F^T and F^Ω for the reasonable range of $|M_{1/2}| \lesssim 10$ TeV. This strong constraint is mainly due to the predictions $m_{H_1, H_2}^2 = -M_{1/2}^2$ and $A_t = -M_{1/2}$ at M_X which are valid for the SUSY breaking by F^T and F^Ω . If another source of SUSY breaking is introduced, e.g. the auxiliary component F^Z and/or $F^{Z'}$ of $\mathcal{Z}, \mathcal{Z}'$ confined at the orbifold fixed point, these predictions are not valid anymore, while the prediction (17) remains to be valid. We examine such general situation in which $A_t/M_{1/2}$ and $m_{H_1, H_2}^2/M_{1/2}^2$ at M_X have arbitrary values of order unity, while the prediction (17) is maintained. For the results depicted in Fig.3, we assumed that the squark and slepton masses at M_X are given by $m_{Q_3, U_3}^2 = (y_t/g_{GUT})^2 M_{1/2}^2$, $m_{D_3}^2 = (y_b/g_{GUT})^2 M_{1/2}^2$ and $m_{E_3, L_3}^2 = (y_\tau/g_{GUT})^2 M_{1/2}^2$, and also $A_t = A_b = A_\tau$. In fact, the results are not so sensitive to m_ϕ^2 ($\phi = Q_3, U_3, D_3, L_3, E_3$), and we obtain similar results as long as $m_\phi^2 = \mathcal{O}(M_{1/2}^2)$.

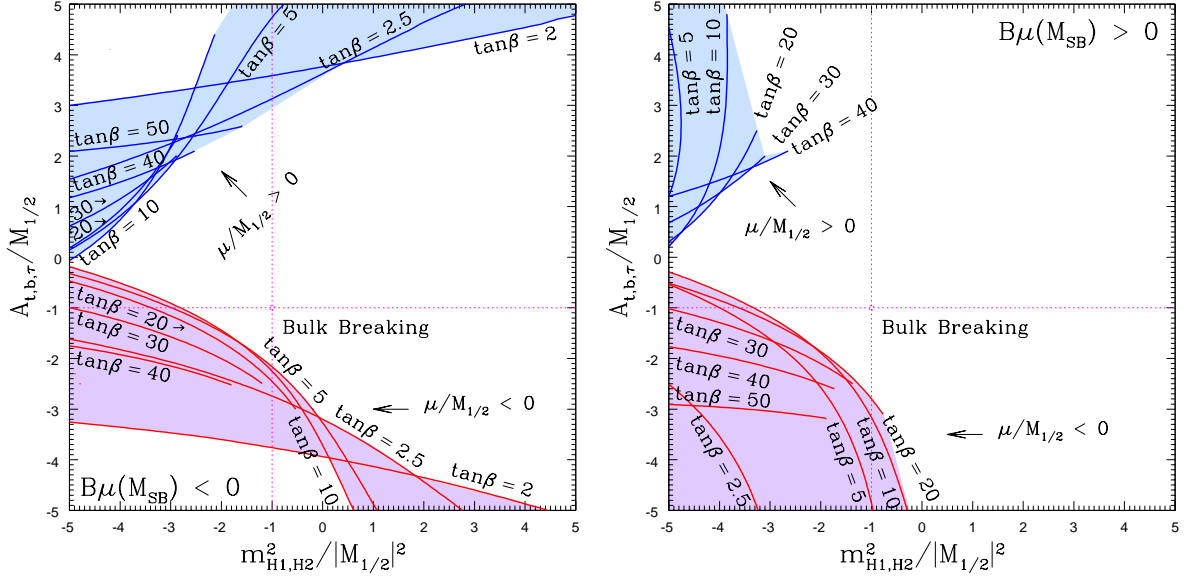


FIG. 3: Shaded parameter regions of $A_t/M_{1/2}$ and $m_{H_1, H_2}^2/M_{1/2}^2$ at M_X give a correct electroweak symmetry breaking vacuum when the SUSY breaking effects from the orbifold fixed points are included for $|M_{1/2}| = 500$ GeV.

The shaded region in Fig.3 represent the parameter region yielding a correct electroweak symmetry breaking for $\tan \beta$ varying from 2 to 60. Each solid curve indicates a contour of fixed $\tan \beta$. Fig.3 shows that if there exists an additional SUSY breaking $F^{\mathcal{Z}, \mathcal{Z}'} = \mathcal{O}(F^T/R)$ or $\mathcal{O}(F^\Omega)$ from the fixed points, sizable parameter region of the model can yield a correct electroweak symmetry breaking. In particular, it shows that any value of $\tan \beta > 2$ can be realized. The point $(m_H^2/|M_{1/2}|^2, A_t/M_{1/2}) = (-1, -1)$ corresponds to the bulk SUSY breaking by F^T and F^Ω , which is obviously outside the shaded regions.

We also performed the analysis for the case that $A_t/M_{1/2}$ and $m_\phi^2/M_{1/2}^2$ at M_X take arbitrary values of order unity, while $m_{H_1, H_2}^2/M_{1/2}^2 = -1$. The results are depicted in Fig.4. Here \bar{m}_ϕ^2 is defined as $(y_{t, b, \tau}/g_{GUT})^2 M_{1/2}^2$, and other parameters are same as in Fig.3, so the point $(m_\phi^2/\bar{m}_\phi^2, A_t/M_{1/2}) = (1, -1)$ corresponds to the SUSY breaking by the bulk F^T and F^Ω . Fig.4 shows that the results are not so sensitive to m_ϕ^2 as long as m_ϕ^2 at M_X is positive. It shows also that to achieve a correct electroweak symmetry breaking with $A_t \approx -M_{1/2}$, a negative stop mass of $\mathcal{O}(M_{1/2}^2)$ is required at M_X , which becomes positive at M_{SB} due to the RG evolution.

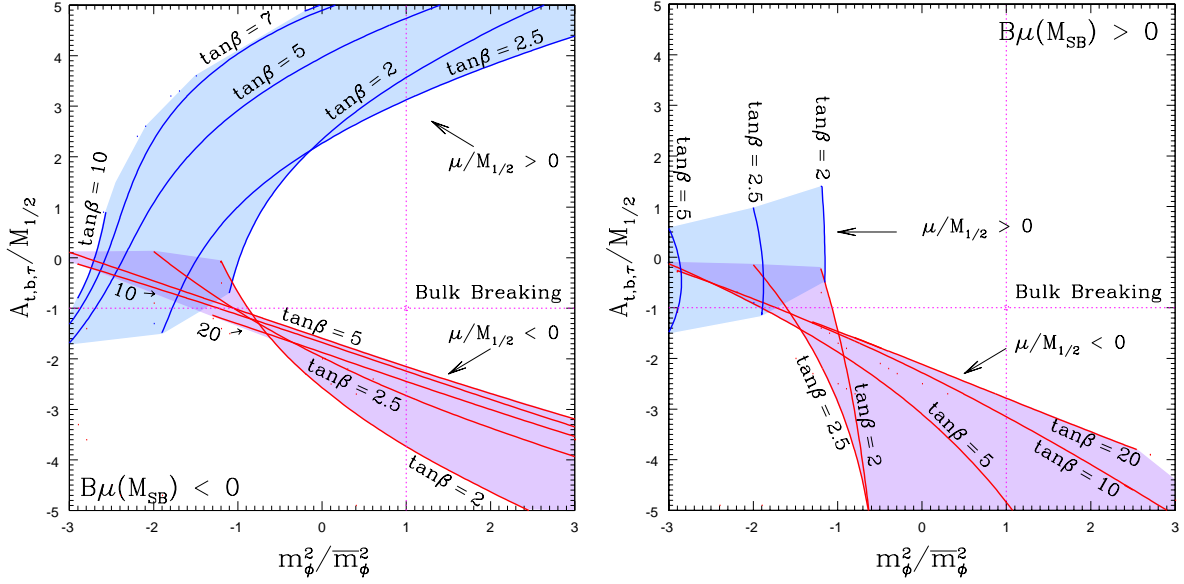


FIG. 4: Shaded parameter regions of $A_t/M_{1/2}$ and m_ϕ^2/\bar{m}_ϕ^2 ($\bar{m}_\phi^2 = (y_{t,b,\tau}/g_{GUT})^2 M_{1/2}^2$) at M_X give a correct electroweak symmetry breaking vacuum when the SUSY breaking effects from the orbifold fixed points are included for $|M_{1/2}| = 500$ GeV.

IV. CONCLUSION

In this paper, we have examined the Higgs mass parameters and electroweak symmetry breaking in supersymmetric orbifold field theories in which the 4D Higgs fields originate from higher-dimensional gauge supermultiplets. To be specific, we focused on 5D models, however some of our results are more generic. It is noted that the gauge-Higgs unification within orbifold field theory leads to a specific boundary condition on the Higgs mass parameters at the compactification scale M_X , which has been obtained also in supersymmetric pseudo-Goldstone Higgs models. More restrictive boundary conditions on the Higgs mass parameters could be obtained when SUSY breaking is mediated dominantly by the auxiliary components of the radion superfield T and the 4D SUGRA multiplet, which corresponds to the Sherk-Schwarz SUSY breaking with additional SUSY breaking from the conformal compensator superfield Ω . If the 4D effective theory at scales below M_X is the MSSM, the SUSY breaking by F^T and F^Ω alone can not give a correct electroweak symmetry breaking vacuum for reasonable range of parameters. So we need additional SUSY breaking mediated for instance

by a brane superfield \mathcal{Z} confined at the orbifold fixed points. If $F^{\mathcal{Z}}$ is included, there exist a sizable portion of parameter space which can give correct electroweak symmetry breaking with wide range of $\tan\beta$. The results of numerical analysis are summarized in Fig.1-Fig.4.

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